

Band in his studies of shock structure.<sup>8</sup> For this latter case it is clear from (17) that  $ds/dV=0$  and, moreover,

$$dP/dV = (\partial P/\partial V)_s + (\partial P/\partial s)_V (ds/dV) = (\partial P/\partial V)_s.$$

#### IV. CONCLUSIONS

We may summarize the conclusions as follows.

(1) The subsonic-supersonic conditions for shocks,  $M_0 \geq 1$ ,  $M_1 \leq 1$  are a consequence of the second law of thermodynamics for viscous, heat conducting fluids with arbitrary equation of state. It is not necessary to invoke the additional conditions,

$$(\partial^2 P/\partial V^2)_s > 0, \quad (\partial P/\partial s)_V > 0.$$

(2) The effect of heat conduction is to reduce the mechanical dissipation in accord with the Le Chatelier-Braun principle.

(3) Under the condition,

$$(\partial P/\partial V)_T < -j^2$$

at the head of the shock, it is not thermodynamically permissible to assume for any material that entropy production is due to heat conduction alone. That is, some viscous dissipation is necessary. This would rule out such predicted phenomena as the "isothermal discontinuity (Ref. 4, p. 342)."

(4) For all materials for which the quantity  $(\partial P/\partial s)_V$  is positive in the shocked state, the entropy must attain a maximum value in the transition region. This conclusion agrees with that derived earlier for an ideal

gas.<sup>5</sup>

These conclusions depend on the correctness of the assumption that the contributions to entropy production due to mechanical dissipation and to heat conduction are individually positive.

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